CROWDSOURCING

The establishment of the ZerOrigIndia Foundation is predicated on a single premise, namely, that our decades-long studies indicate that there are sound reasons to assume that facilitating further independent scientific research into the origin of the zero digit as numeral may lead to theoretical insights and practical innovations equal to or perhaps even exceeding the revolutionary progress to which the historic emergence of the zero digit in India somewhere between 200 BCE and 500 CE has led across the planet, in the fields of mathematics, science and technology since its first emergence.

No one to date can doubt the astounding utility of the tenth and last digit to complete the decimal system, yet the origin of the zero digit is shrouded in mystery to this day. It is high time, therefore, that a systematic and concerted effort is undertaken by a multidisciplinary team of experts to unearth any extant evidence bearing on the origin of the zero digit in India.

The ZerOrigIndia Foundation is intended to serve as instrument to collect the requisite funds to finance said independent scientific research in a timely and effective manner.

Research

Academics and researchers worldwide are invited to join our efforts to unearth any extant evidence of the zero digit in India.

The **ZerOrigIndia Foundation** will facilitate the research in various ways, chief among which is to engage in fundraising to finance projects related to our objective.

Academics and researchers associated with reputed institutions of higher learning are invited to monitor progress reported by **ZerOrigIndia Foundation**, make suggestions and/or propose their own research projects to achieve the avowed aim.

Science Committee

ZerOrigIndia Foundation also invites academics and researchers to sit on our Science Committee in order to assess for approval research proposals from their peers to be conducted under **ZerOrigIndia Foundation**'s auspices.

Ethics Committee

ZerOrigIndia Foundation also invites academics and researchers to sit on our Ethics Committee in order to enforce strict ethical standards applicable during the course of research conducted in pursuit of the objective.

Introduction

Human beings have been counting for tens of thousands of years. But it is only within past millennia that mathematics proper has been practiced in advanced civilizations.

Place-value systems, including base-10, base-20, base-60 and others, are found early on, with or without a zero placeholder, that is, a symbol to represent the absence of a digit in a number. Numeration systems with a placeholder zero were in use for hundreds of years without the symbol representing zero being used as actual numeral in its own right.

It is widely agreed that the use of the zero digit not only as placeholder but also as fully-fledged numeral alongside the other numerals was an unrivaled innovation that revolutionized mathematics, science and technology. It is stating the obvious to note that practically all nations on Earth have since adopted the decimal system including zero, while few people today are aware where the numbers they use daily came from.

And while there is general agreement among scholars that the decimal system ubiquitously in use today hails from India, reaching medieval Europe via the Arabs, there is no consensus as to whether or not the zero digit was an indigenous South Asian invention or whether it was 'imported' – either from points further to the West or the East.

Thus there is much conjecture and controversy surrounding the emergence of the zero digit as numeral and to this day it has never been incontrovertibly established precisely where and when or in what cultural context the zero digit first appeared, let alone who the individual was to be credited with its invention - which has been ranked among mankind's greatest intellectual feats.

The default view on the otherwise inexplicable emergence of the zero digit in history may be characterized as a variation on the 'necessity is the mother of invention'-notion. That is to say, when the need arose for zero, someone came up with it.

However, on second thought it is rather a tautology with little or no explanatory power. Did the need for zero arise and then the zero digit was invented; or was the zero digit invented when it was considered that the need had arisen?

Had the need not arisen hundreds and even thousands of years earlier when advanced civilization were already doing complex mathematical and astronomical calculations and thus could have used the zero digit to advantage? Why was the zero digit not invented earlier and elsewhere than somewhere on the Indian subcontinent between 200 BCE and 500 CE?

The minority view as to why the zero digit arose is related to the cultural / philosophical wherewithal at the disposal of early mathematicians; or contrariwise, the absence of the zero digit owing to the absence of such a tradition enabling thinkers to even conceive of 'absence' or 'emptiness' not just in terms of quality, but in terms of quantity as well.

Why India?

Studies conducted by the **ZerOrigIndia Foundation** discern the following slow evolutionary trajectory in Indic civilization from the pre-Vedic period, to the Vedas, the Upanishads and via certain of the orthodox and heterodox philosophical systems, centering on the pivotal concept of śūnya/purna or emptiness/fullness, which in the final analysis manifested in a range of cultural expression in mysticism, religion, prose, poetry, visual arts, music and even architecture. In that sense the otherwise inexplicable emergence of the mathematical equivalent of śūnya/purna concept in the form of the zero digit and its obverse infinity were rather latecomers.

While to this very day the issue of India's claim to be the cradle of the mathematical zero is not uncontested among scholars, there is much circumstantial evidence to justify the claim, which to be fully convincing does require further research to be conducted by a well-funded and staffed multidisciplinary team of scholars and researcher in order to unearth any extant evidence bearing on the issue.

What follows is an abridged list of recurrent references in the literature to support India's claim to the innovation of the zero digit, without ever any mention made of recent, on-going or planned research to unearth fresh evidence:

- The Sanskrit language and the Vedas were predisposed towards the decimal system, containing frequent mention of multiples of ten, a hundred, a thousand et cetera up to stupendously large numbers;
- Pāṇini's grammatical system applied the concept of lopa, or absence;
- Pingala's metrics actually used the term śūnya for zero in what was a form of binarism;
- Ancient Jaina works used decimal place-value;
- The Bakhshali manuscript used the zero digit as numeral (of uncertain date, estimates ranging from 3rd century BCE – 7th century CE and even later);
- Aryabhata I (5th century CE), famous astronomer-mathematician, used positional decimals;
- Brahmagupta (7th century CE), mathematician credited with defining zero and its operations in the *Brāhmasphuṭasiddhānta* (628)
- Earliest inscription of zero digit in India at Gwalior temple (876 CE)
- Rival claims are often mentioned such as the Khmere inscription at Sambor (683 CE) and the Malay inscription at Palembang (684 CE), it being pointed out variously that these were 'Indianised' colonies at the time;

The crucial distinction must throughout be borne in mind between a positional decimal system with zero as 'mere' placeholder, and zero as fully fledged numeral alongside the other 9 digits. Some of the mathematical texts cited in the literature refer to examples of both without acknowledging this crucial distinction and yet claiming zero's innovation as digit for India or for that matter other contenders. The research proposal is intended to clear up, among others, precisely this issue regarding India.

This defining element of the zero concept is stated by Robert Kaplan in his book *The Nothing That Is; A Natural History of Zero* (1999), p. 45. It, too, is consistent with the **ZerOrigIndia Foundation** working hypothesis that the mathematical concept of zero arose out of a sophisticated Vedic philosophical tradition -- though Kaplan does not acknowledge this. Moreover, as he makes abundantly clear throughout, zero as placeholder is not seen by him as indigenous to India while seeming to concede it to be the birthplace of the zero digit as numeral. The question as to when, where or by whom is not seriously pursued.

Writes Kaplan: "Once again, while having a symbol for zero matters, having the notion matters more, and whether this came from the Babylonians directly or through the Greeks, what is hanging in the balance here in India is the character this notion will take: will it be the idea of absence of any number – or the idea of a number for such absence? Is it to be the mark of the empty, or the empty mark? The first keeps it estranged from numbers, merely part of the landscape through which they move; the second puts it on a par with them."

Be that as it may, the intellectual climate of the day in South Asia readily accommodated the idea that 'everything comes from nothing', the requisite 'template', if you will, that gave rise to the *definition of zero by Brahmagupta in the 7th century CE as the sum of equal numerals of opposite sign*. In fact, this *creatio ex nihilo* notion may have spawned the idea that all numerals find their origin and end in zero – quite akin to the thrust of modern set theory well over a thousand years hence. And with it the concept of negative numbers as well.

Contrariwise it would account for the fact that neither the pragmatic Greeks nor for that matter any other civilization could avail of the zero digit as fully-fledged numeral alongside the other numerals, as a sophisticated notion of non-trivial emptiness would have been a necessary - but not necessarily sufficient - prerequisite. This in contradistinction to the 'mere' placeholder zero, which while brilliant in its own right is not barred from human conception by fundamental religio-philosophical constraints.

Consider the surmise by the renowned 18th-century French astronomer and mathematician, Pierre-Simon Laplace:

"It is India which gave us the ingenious method of expressing all numbers by means of ten symbols, each symbol receiving a value of position as well as an absolute value, a profound and important idea which appears so simple to us now that we ignore its true merit. But its very simplicity, the great ease which it lent to all computation, puts our arithmetic in the first rank of useful inventions and we appreciate the grandeur of this achievement the more, when we remember that it escaped the genius of Archimedes and Apollonius, two of the greatest men produced by antiquity."

Consider, for example, the invocation to the Isha Upanishad (circa 800-500 BCE), which reads:

"That is full; this is full. The full comes out of the full. Taking the full from the full the full itself remains."

- *The Principal Upanisads*, p. 565, by S. Radhakrishnan, Harper Collins Publishers India (2003)

In hindsight the modern reader may recognize in this invocation, rather than mystical gibberish, a template for the much later innovation of the zero digit (śūnya), and its obverse, infinity. No other digit would fit the bill, so to speak. It would have fallen to a perspicuous mathematician of the day, steeped in the 'spirit of the time' [Buddhist philosophy Emptiness or Śūnyatā, 200BCE-500CE] to make the connection.

Inter alia, again in hindsight the invocation also appears to imply the mathematical operation of subtraction, or negation, also implicit in the *creation ex nihilo* concept. One would be justified to see the invocation as a perspicuous template for developments to come – evolution in progress, as it were.

For the sake of convenience we refer here to an excellent 'primer' on the subject of zero's literary and philosophical antecedents in India in an article by Dr. Parthassarathi

Mukhopadhyay, *Concept of Śūnya In Indian Antiquity*, obtained indirectly from him as email attachment and included here for the purpose of ready reference.

[excerpt]

Abstract

The genesis of "zero" as a number, that even a child so casually uses today, is a long and involved one. A substantial number of persons concerned with the history of its evolution, today accept that the number "zero", in its true potential, as we use it in our present day mathematics has its root, conceptually as well as etymologically, in the word "Śūnya" of Indian antiquity, and it was introduced in India by the Hindu mathematicians. This eventually became a numeral for mathematical expression for "nothing", and via the Arabs, went to Europe. The time frame of its origin in Indian Antiquity is still hotly debated. One recent suggestion, from some astronomical calculations is that, it probably appeared in 458 CE as found in the Jaina cosmological text Lokavibhaga (meaning, The Parts of the Universe). Furthermore, some recent works even try to suggest that a trace of the concept, if not in total operational perspective, might have a Greek origin that traveled to India during the Greek invasion of the northern part of the country in the pre-Mauryan period.

In this article we would like to discuss the available references to the concept of "Śūnya" or its numerous synonyms in its broader social and philosophical contexts as was used in Indian antiquity, which eventually paved the path for the evolution of the corresponding mathematical concept. From the works on Vedic prosody by Pingala (Chandaḥsūtra) [3rd Century BC] to the concept of "lopa" in the grammarian Pāṇini. (Aṣṭādhyāyī) (sometime between 700-400 BCE, by some modern estimates) it appears very likely that the thread of rich philosophical and socio-academic ambiences of Indian antiquity was quite pregnant with the immensity of the concept of "Śūnya"—a dichotomy as well as a simultaneity between nothing and everything, the "zero" of void and that of an all pervading "fathomless" infinite.

The case of *Sphujidhvaja's Yavanajātaka*

History of Science in South Asia 1 (2013): 1-20
Online version available at: http://hssa.sayahna.org/.

The Date and Nature of Sphujidhvaja's Yavanajātaka Reconsidered in the Light of Some Newly Discovered Materials

Bill M. Mak
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[excerpts]

[…] This paper examines a number of crucial verses from the last chapter of Sphujidhvaja's Yavanajātaka, a text that was widely believed to be the earliest Greek astral text translated into Sanskrit. In the light of some new materials, including a hitherto unreported Nepalese paper manuscript from the collection of the Nepal-German Manuscript Preservation Project (NGMPP), the date and nature of this important text are reconsidered. […] While this does not mean to deny the claim of the discovery of zero as number in India, evidences for such mathematical innovation must be sought elsewhere as some scholars have attempted. […] In 2011-2012, additional materials including a hitherto unreported copy of the Javanajātaka became available to the present author. […] In 2011, I was informed by Michio Yano of his discovery of a hitherto unreported copy of the Yavanajātaka which was mistakenly recorded as (Bṛhad)-yavanajātaka in the NGMPP database. A digital set of color photos was produced (Q). Upon examination, this manuscript is found to contain significant readings which fill up some lacunae of our copies of N, as well as with significant variants. The manuscript contains 78 pages and was numbered up to 90, with therefore 12 pages missing. While this manuscript could be as late as the eighteenth century judging from the paper quality, it provides us also some additional information about the text which was not available in N. […]The "incorrectly written manuscript" N (folios 2–103) used by Pingree was in fact a microfilm of a Nepalese manuscript now in the possession of the National Archives in Nepal.

For the purpose of the present **ZerOrigIndia Foundation** research proposal's Position Paper, please note the important late discovery of a previously archived ancient manuscript that had not yet been studied for evidence of the early appearance of zero. That the outcome of Dr Mak's research furthermore in his own words 'refuted' the earlier findings of an acknowledged authority on the subject, the late Professor David Pingree, is no less significant in this context.

It may serve as encouragement to scholars seriously interested in taking up the research project as the case under review here is a vivid reminder that crucial, previously undiscovered evidence of the origin of the zero digit in India may as yet be extant.

FURTHER RESEARCH

India then would appear eminently qualified to stake its claim to being the cradle of the fully-fledged zero. Given the uncertainty to this very day as to the venue of the birth of the zero digit, which has so altered the face of the Earth since its arrival on the scene roughly two millennia ago, a concerted effort in the form of a well-funded and well-staffed multidisciplinary research team to establish zero's pedigree would seem warranted. The more so since such a search may simultaneously shed light on separate but related issues of a non-mathematical nature in related fields such as palaeography, archaeology, epigraphy, numismatics, anthropology, religion, philosophy, linguistics and others.

Circuitously then, the śūnya/purna concept, with its roots reaching deep into the pre-Vedic tradition of South Asia's Indic civilization, flowered again on young Europe's soil,

producing comparably salubrious fruits in the form of unprecedented scientific progress, culminating in state-of-the-art quantum physics, which finds itself overwhelmed by its own discoveries that so clash with its Greek-Christian heritage.

Consider the following surmise: "Physicists have grappled with the quantum world's apparent paradoxes for nine decades, with little to show for their struggles. […] quantum theory is still considered (even by many physicists) to be a bizarre anomaly, a powerful recipe book for building gadgets but good for little else. The deep confusion about the meaning of quantum theory will continue to add fuel to the perception that the deep things it is so urgently trying to tell us about our world are irrelevant to everyday life and too weird to matter."

- *Quantum Weirdness? It's All in Your Mind*, by Hans Christian von Baeyer, Scientific American, June 2013, p.39/40.

In the late 20th century, the light finally began to dawn when erudite quantum physicist Dr Fritjof Capra, and others as well, conversant with Indian and other ancient traditions, recognized the uncanny convergence of worldviews that emerged from their pioneering work and, for example, Hinduism and Buddhism.

But while Capra was at a loss to provide ***a causal explanation*** for this surprising insight, it could only be accounted for in terms of mere coincidence. However, it makes perfect sense, and be not just a curious coincidence, if one bears in mind that the causal connection could be accounted for in terms of ***the 'semiotic' power packed by the decimal system including the zero that was transmitted across hundreds of years and thousands of kilometers from India to the West.***

The salient feature of semiotics involved is that objects of thought are not 'anterior' to the signs or symbols that represent them but rather the other way around. In that way the arrival of the zero digit and the concept of emptiness heralded access to whole new realms of thought, including mathematical thought, the human mind being a symbolific instrument making sense of its environment, in the words of Aldous Huxley.

That transmission and diffusion of cultural assets are a 2-way street may be illustrated by a quote from Capra [available on Internet], who had the privilege of interviewing one of the pioneers of quantum theory, the late-Werner Heisenberg, who formulated the famous 'Heisenberg Uncertainty Principle':

"I had several discussions with Heisenberg. I lived in England then [circa 1972], and I visited him several times in Munich and showed him the whole manuscript chapter by chapter. He was very interested and very open, and he told me something that I think is not known publicly because he never published it. He said that he was well aware of these parallels. While he was working on quantum theory he went to India to lecture and was a guest of Tagore. He talked a lot with Tagore about Indian philosophy. Heisenberg told me that these talks had helped him a lot with his work in physics, because they showed him that all these new ideas in quantum physics were in fact not all that crazy. He realized there was, in fact, a whole culture that subscribed to very similar ideas. Heisenberg said that this was a great help for him."

Core concepts in quantum mechanics, such as *uncertainty* and *indeterminacy* are also linked to Indian philosophy by the late-Dr David Seyfourt Ruegg, one of the stalwarts of Buddhist canon and Madhyamaka texts in particular:

"The indulgence of some readers will no doubt be required when there is introduced into the discussion a pair of concepts – indeterminacy and uncertainty – that may be reminiscent of the language which has been used for describing the theoretical foundations of quantum physics…And it is not being suggested here that Madhyamaka thought was somehow an anticipation or precursor of quantum theory. Yet, in regard to the conditioned origination of things on the relative, transactional-pragmatic level, Madhyamakas do in their own way have a sense of what might be called the probabilistic, as well as of the impossibility of setting up any unified theory of things in the frame of an ontology predicated on the substantial existence of reified entities…the Madhyamaka thinker has regularly worked within the frame, not of entities conceived of as produced fortuitously but, rather, of things – i.e. phenomenal events – thought of as originating in dependence on causes and conditions. These are accordingly described as Empty of self-existence…Nonetheless, the descriptive language once employed by Werner Heisenberg – invoking in quantum physics the principle of indeterminacy (Unbestimmtheitsrelation) or uncertainty (Unsicherheitsrelation), and also dependence of scientific observation and their measurement on the experimental procedures being adopted by the observer (the physicist) himself – would seem to lend itself, mutatis mutandis, to describing (if only by analogy and approximation) certain ideas met with in Madhyamaka thinking, notwithstanding the clear differences in their respective problematics and contents."

- ***The Buddhist Philosophy of the Middle, Essays on Indian and Tibetan Madhyamaka***, by David Seyfort Ruegg, Wisdom Publications, Boston (2010)

It could thus account for the astounding convergence of worldviews emerging from modern physics, and that of India's philosophic traditions, as noted by prominent Western scientists such as Fritjof Capra. ***But whereas earlier no causal relationship was suggested to account for this convergence of worldviews, the main working hypothesis to emerge from the present research proposal is that this convergence between the two otherwise distinct traditions may be due to the decimal system plus zero that they share in common, since zero signifies the concept of non-trivial emptiness, and the syntax relating the ten digits is alien to the West (please see below).***

In an article titled *To What Type of Logic Does the "Tetralemma" Belong?,* Professor Raphael D. Sorkin writes:

"Considered from the standpoint of classical logic, the fourfold structure of the so-called tetralemma (catuṣkoṭi) appears to be irrational, and modern commentators have often struggled to explain its peculiar combination of alternatives […] A possible answer comes from quantum mechanics, where certain alternative logics have been proposed as a solution to the paradoxes that arise in the attempt to describe ***subatomic reality*** *[stress added, PG]. In the early proposals of this sort, known collectively as "quantum logic", the laws for combining propositions were modified in such a way that the distributive law no longer holds. More recently though a different type of logical structure has been put forward in which the rules for combining propositions are classical ones but what changes are the rules of inference. It is these "anhomomorphic" logics, I would suggest, that hold the key to understanding the catuṣkoṭi form. […] To the extent that Indian thinkers in the time of Gotama were aware of this possibility, they would naturally have phrased their questions in*

"tetralemmatic" form. […] It would be interesting to know what led ancient thinkers to recognize – if they did – the possibility of an anhomomorphic logic. They cannot have had access to the kind of technology that has led in modern times to quantum physics. Are there then other experiences that one could point to which were in fact available to them and to which anhomomorphic inference is more suited that homomorphic inference? If so, we might gain a better intuition for the micro-world by ourselves paying more attention to those experiences."

One of the key lessons to be drawn from Sorkin's article, which is not made explicit, is that the so-called 'classical homomorphic logic' referred to is Aristotelian 2-value logic, again a legacy of the Greeks, that while useful under certain circumstances, hamstrung Western thinkers as an insidious tacit assumption warping their take on things and thus prejudicing zero's reception when the time came.

The Greek tradition had also saddled up the Western tradition with the notion that creation out of nothing, *creatio ex nihilo*, was considered impossible, later even proscribed by the Christian Church, and as such hobbled science in the West for centuries. It was only after the Hindu decimal system, including zero, was introduced in 13th-century Europe that horizons were expanded and progress made possible.

By the same token, zero's "ghostly underbelly", as Emeritus Professor Brian Rotman famously phrased it in private email correspondence, also seems very much present in our Information Age, as in quantum physics. According to him [*Signifying Nothing; The Semiotics of Zero*: Stanford University Press (1987), p.105/107:

"The sign 1 can be anything. If 1 is one and 0 is zero and the syntax is the standard positional notation for numbers, then what results, as a limiting case of the system, is the two-valued descendant of the Hindu decimal system. Leibniz, who spent much time formulating the rules for binary arithmetic, was deeply impressed by the generative, infinitely prolific principle inherent in such a zero-based binarism: so much so that he refracted the binary relation between 1 and 0 into an iconic image for the Old Testament account of creation ex nihilo, whereby the universe (the infinitude of numbers) is created by God (the unbroken 1) from the void (the cypher 0). Again, if 1 signifies the presence of a current and 0 signifies the absence of such a current and the syntax is 2-valied Boolean algebra, then what emerges is the binary formalism within which the logic and language of all present-day computer programs are ultimately written."

Addressing both these apparently related enigmas in physics and computer science that surface in relation to the application of the non-classic logic unwittingly imported from India together with the decimal system including zero, Dr C.K. Raju observes in his paper available on Internet, quoted here at some length:

"By way of background I point out that since the theorems of formal mathematics vary also with logic, and since logic varies with culture (e.g. neither Buddhist nor Jaina logic is 2-valued), and is also not empirically certain (e.g. quantum logic), formal proof (or deduction) can never aspire to be either certain or universal (or even more certain than induction), contrary to the popular philosophical and cultural beliefs in the West. The origin of these popular Western beliefs about mathematical proof can be readily understood through the historical evolution of the Egyptian/Neoplatonic ("Euclidean") notion of proof, as modified to suit the requirements first of Islamic rational theology, and then of Christian

rational theology, and later incorporated into formal mathematics by Hilbert, Russell etc. by modifying the Elements. The Indian notion of pramāṇa, in contrast, rooted in an empirical and practical understanding of the world, did not view mathematics through a cultural understanding in which metaphysics ("mathematical truth") was placed on a higher plane than physics.

"These two views of mathematical proof were brought into confrontation with each other when Indian calculus travelled to Europe. The European difficulty with zero did not concern merely the numeral zero, but related also to the process of discarding or zeroing a "non-representable" during the course of a calculation—similar to the process of rounding. Though the Indian method of summing the infinite series constituted valid pramāṇa it was not understood in Europe; the earlier difficulty with non-representables zeroed during a calculation reappeared in a new form. This was now seen as a new difficulty—the problem of discarding infinitesimals. Berkeley's criticism makes it obvious that up to the time of their death neither Newton nor Leibniz had reconciled their methods of dealing with fluxions/infinitesimals with the clarity expected of them by their contemporaries. (Retrospective disambiguation of Newton and Leibniz is as irrelevant as retrospective disambiguation of the prophecies of, say, the Oracle of Delphi or Nostradamus.) In both cases of algorismus and calculus, Europeans were unable to reject the new mathematical techniques because of the tremendous practical value for calculations (required for commerce, navigation etc.), and unable also to accept them because they did not fit in the metaphysical frame of what Europeans then regarded as valid.

"Any stable way out of these present-day difficulties must address the central thread common to the thousand-year old European difficulties with arithmetic, calculus, and now computers—the problem of non-representability. A simple way out is to revert to the original philosophy of non-representables, as implicitly used in the Śulbasūtras, as inherent in Indian arithmetic, trigonometry, and calculus, and as explicitly articulated in Nāgārjuna's śūnyavāda) ("zero-ism", Zen). This would also help to resolve the problem with representability made manifest by computers. I argue that this way out is to be preferred to repeatedly forcing mathematics to fit into Platonic idealism, and related religious beliefs, inherently contrary to mathematics-as-calculation, merely on the basis of a largely mythical and racist view of history which locates the origins of mathematics in "Greece". Such a change in the philosophy of mathematics would also be most appropriate to the changes in the understanding of mathematics and number that may be expected to accompany the more recent enhancement in the capability to calculate, using computers."

- C. K. Raju, "The Mathematical Epistemology of Śūnya", interventions during the Seminar on Concept of Śūnya, Delhi, 1997, in: The Concept of Śūnya, ed. A. K. Bag and S. R. Sarma, IGNCA, INSA, Delhi, 2002, 168–81. The Indian Origins of the Calculus and its Transmission to Europe Prior to Newton and Leibniz. Part II: Lessons for Mathematics Education, C. K. Raju , Centre for Studies in Civilizations, New Delhi & Centre for Computer Science, MCRP University Bhopal

Non-mathematical benefits to be derived from the research proposal on the origin of zero in India.

Finally, then, a targeted search for zero's origins in India would almost inevitable lead to serendipity collateral discoveries of interest to fields other than mathematics, science and

technology, and as such serve as additional fillip in favour of undertaking the investigation of India's as yet underexposed cultural heritage to unearth early appearances of zero in India.

Discussion

Advanced civilizations have for millennia practiced mathematics and astronomy without the benefit of a zero digit on a par with other numerals. Apparently the diverse numeral systems applied worldwide met their requirements without leading to the innovation of the zero digit. The default argument in some quarters to account for the innovation of the zero digit is that when the need arose due to complex mathematical calculations, the zero digit was invented.

However, such a 'necessity-is-the-mother-of-invention' hypothesis is not credible as otherwise the zero digit would have surfaced earlier and elsewhere than in India, which it arguably did not.

But even its emergence in India begs the question as to why there and then? Generally speaking, while all and sundry pay the highest tribute to the zero digit (please see Appendix I), few ask the question as to how or why the zero digit was invented. And those who do proffer suggestions (Appendix II), do not propose any research to substantiate the hypothesis.

It is for these and other reasons that the **ZerOrigIndia Foundation** proposes to facilitate the launch of a research project in order to attempt to discover what evidence may still be found to resolve this vital issue roughly two millennia after the fact.

*

APPENDIX I

WHAT PEOPLE HAVE SAID ABOUT ZERO

Pierre-Simon Laplace (1749–1827), astronomer and mathematician	"It is India which gave us the ingenious method of expressing all numbers by means of ten symbols, each symbol receiving a value of position as well as an absolute value, a profound and important idea which appears so simple to us now that we ignore its true merit. But its very simplicity, the great ease which it lent to all computation, puts our arithmetic in the first rank of useful inventions and we appreciate the grandeur of this achievement the more, when we remember that it escaped the genius of Archimedes and Apollonius, two of the greatest men produced by antiquity."
Prof. G.B. Halstead, *On the foundation and technique of Arithmetic*, Chicago, 1912 p.20	"The importance of the creation of the zero mark can never be exaggerated. This giving to airy nothing, not merely a local habitation and a name, a picture, a symbol, but helpful power, is the characteristic of the Hindu race whence it sprang. It is like coining the Nirvana into dynamos. No single mathematical creation has been more potent for the general on-go of intelligence and power."
E.T. Bell (1883-1960), mathematician and science fiction author	"The problem of numeration was formally solved by the Hindus at some controversial date before 800 A.D. The introduction of zero as symbol denoting the absence of units of certain powers of ten in a number represented by the Hindu numerals has been rated as one of the greatest practical inventions of all times."
Robert Kaplan, *The Nothing That Is; A natural History of Zero*, 1999, p.1	"If you look at zero you see nothing; but look through it and you will see the world. For zero brings into focus the great, organic sprawl of mathematics, and mathematics in turn the complex nature of things. From counting to calculating, from estimating the odds to knowing exactly when the tides in our affairs will crest, the shining tools of

	mathematics let us follow the tacking course everything takes through everything else – and all of their parts swing on the smallest of pivots, zero."
Dr George Gheverghese Joseph, history of mathematics professor, *The Crest of the Peacock; Non-European Roots of Mathematics,* 2011, p.344/5	[Section:] <u>The Enormity of Zero</u> "In India, zero as a concept probably predated zero as a number by hundreds of years. The Sanskrit word for zero, śūnya, meant 'void' or 'empty'. The word is probably derived from shuna, which is the past participle of svi, 'to grow'. In one of the early Vedas, Rig-veda, there is another meaning: the sense of 'lack' or 'deficiency'. It is possible that the two different words were fused to give śūnya a single sense of 'absence' or 'emptiness' with the potential for growth. Hence its derivative, Śūnyata, described the Buddhist doctrine of 'Emptiness,' being the spiritual practice of emptying the mind of all impressions. This was a course of action prescribed in a wide range of creative endeavors. For example, practice of Śūnyata is recommended in writing poetry, composing a piece of music, producing a painting, or any activity that comes out of the mind of the artist. An architect was advised in the traditional manuals of architecture (the Silpas) that designing a building involved the organization of empty space, for 'it is not the walls that make a building but the empty spaces created by the walls'…**The mathematical correspondence was soon established** (italics added). 'Just as emptiness of space is a necessary condition for the appearance of an object, the number zero being no number at all is the condition for the existence of all numbers.'"
Charles Seife, *Zero; The biography of a dangerous idea,* 2012, p.2/3 and p.66	"This is the story of zero, from its birth in ancient times to its growth and nourishment in the East, its struggle for acceptance in the Europe, its ascendance in the West, and its ever-present threat to modern physics…Yet through all its history, despite the rejection and the exile, zero has always defeated those who opposed it. Humanity could never force

	zero to fit its philosophies. Instead zero shaped humanity's view of the universe – and of God." "So India, as a society that actively explored the void and the infinite, accepted zero…Indian mathematicians did more than simply accept zero. They transformed it, changing its role from mere placeholder to a number. This reincarnation was what gave zero its power."
R.C. Gupta, *Technology of using Śūnya in India*, *The Concept of Śūnya*, 2003, p.23	"Anyway, we find that the concept of Śūnya or zero provoked ideas which engulfed almost all fields of human endeavour. It played a significant role in arts, sciences and other fields not only in India but in the whole world. The story of the spread and triumph of the Indian decimal place-value system with zero is indeed marvellous because it revolutionised the scientific development all over the globe. It was also a cultural victory for India brought not by sword but by merit of the case."
Shri Ajay Mitra Shastri, *Brahmi Numerals and Decimal Notation, The Concept of Śūnya*, 2003, p.74	"…and some genius invented the zero or Śūnya which made arithmetical calculations extremely simple…When the zero came to be invented is at present impossible to determine owing to a total absence of evidence on the point…Whatever that be, the fact remains that it is not found in epigraphs for a considerably long time where the old numerals still continue to reign.
S.R. Sarma, *Śūnya in Pingala's Chandaḥsūtra, The Concept of Śūnya*, 2003, p.131-3	"To conclude, Pingala's mention of Śūnya is a significant event in the history of ideas. It shows that decimal place-value with numbers 1 to 9 and zero developed in India before the beginning of the Christian era."
A.K. Bag, *Śūnya in Pingala's Chandaḥsūtra, The Concept of Śūnya*, 2003, p.159	"The experience of Śūnya or void on a theoretical or philosophical plane is perhaps common with all the human cultures. I personally feel that it is nothing special of Indian culture. My main objective in this presentation is to assess whether there was any situation where Śūnya as a part of the numerical system was conceived to tackle problems or simplify a system. Georg Sarton (1955), the famous historian of science, observes: 'Our numerals and the use of zero

	were invented by the Hindus and transmitted to us by the Arabs.' Different facets to the problem have already been presented here, but I find no such situation or compulsion needed for such a discovery. On this background I wish to re-examine the whole set of evidences starting from the Vedic period."
P. Manansala, *Number Mysticism in Other Regions and the Impact of Śūnya, The Concept of Śūnya*, 2003, p.239/40	"In ancient times, the mathematician, in addition to often being astronomer, navigator etc., also played the part of priest and philosopher. Even today…Considering that philosophers often see the greater cosmos harmonically mirrored in microcosm, the mysterious world of numbers with its series, progressions, harmonies etc., would naturally attract their curiosity and study. The beginning, or creation of the cosmos is one of the mysteries that has most intrigued the human mind. In relating the system of numbers to this beginning, the most common starting point before the advent of Śūnya was naturally the number one. What could better represent the first cause. The number two, it follows, would naturally represent the beginning of duality. There is a weakness in this system regarding cosmological views that view the beginning as essentially empty, or as an absence of objectivity. Eastern systems such as those of the Upanishads, Buddhism and Taoism essentially had such an outlook. The idea of non-duality, monism, Śūnyatā and similar concepts were well-suited, in regard to number mysticism, to the number Śūnya, or zero. Whether such philosophies, which predate the first recorded instances of the use of Śūnya, had any influence on the development of the latter is beyond the scope of this work. Certainly it is possible, but whatever the case, the creation of a symbol for Śūnya could not have found a better suited environment in which to arise."
Staal, F., 2010, "On the Origins of Zero." In *Studies in the History of Indian Mathematics.* Edited by C.S. Seshadri. New Delhi:	"The *Srauta Sutras*, late Vedic but pre-Buddhist, used lopa to refer to omissions, disappearances and things that are lost. It is

Hindustan Book Agency, pp.50/51.	here that the origins of the mathematical concept of zero seem to lie. We do not know where it happened if it happened only once, but the most likely place would be the Kuru region north of modern Delhi though it may have been further to the east in Magadha, which overlaps with modern Bihar. The time must have been 1,000 and before 600 BCE when the creative period of the Srauta ritual was over. It is a long period with smudgy edges but there it is. "As far as I can tell, thus far, two conditions must be satisfied before a concept of zero may arise: there needs to be a language as well as another formal structure in which that language is used to signify that something has disappeared or is lost.

APPENDIX II

THE ORIGIN OF ZERO IN INDIA
WORKING HYPOTHESIS
ŚŪNYATĀ AND ZERO

The Buddhist Philosophy of the Middle, Essays on Indian and Tibetan Madhyamaka, by David Seyfort Ruegg, Wisdom Publications, Boston (2010) – excerpts:	
p.4/5	"In view of the fact that in the Sanskrit mathematical and astronomical literature the word Śūnya has the meaning of zero, there exists another point in the history of Indian thought where, at least in principle, it might be supposed that mathematics has had some influence on philosophy: the Buddhist theory of emptiness (Śūnyatā) of dharmas. And some scholars have in fact suggested a connexion between the Madhyamaka theory of Śūnyatā developed by Nāgārjuna and the mathematical zero. "It is, however, to be noted in the first place that Śūnya as a term for zero appears much later in our sources that the canonical Buddhist concept, and also after Nāgārjuna (no later than 200 C.E.)."
P.90, footnote 154	"It should be noted, however, that there is no evidence in the basic texts of the Madhyamaka school that a mathematical model (and place value) had any immediate bearing on their theory of Śūnyatā. In the Madhyamaka the term Śūnya refers to the fact that any dharma is empty of own-being (svabhava śūnya), in which notion there is no mathematical connotation."
p.92/93, Ruegg quoting 1958 English translation of 1954 work by the Japanese	"'It must be permitted after all to indicate Voidness by "0"" for in two-value logic

logician N. Nakamura, dealing with the formalization of Buddhist logic by means of symbolic logic. p.95, quoting R.H. Robinson on Nāgārjuna	"'all that does not exist is expressed by "0" and is called the "null class", indicating something that cannot exist and expressing falsehood'". "The Śūnyavada is in fact a kind of theory of fictions. The concept of designation (prajnapti) provides a way of handling abstracts without concretizing them, or assigning ontological value to them."
The Concept of Śūnya (2003), Edited by A.K. Bag and S.R. Sarma, Aryan Books International, New Delhi -- excerpts:	
A.K. Bag and S.R. Sarma, p.viii Introduction	"It is also believed that the philosophical conceptualization of Śūnya and Śūnyatā as developed by Nāgārjuna and of Ākāśa of Kanada lent support to and made an impact on the conceptualization of Śūnya of the mathematicians."
Technology of Using Śūnya in India, by R.C. Gupta, p.19 Zero in the Mathematical System of India, by R.C. Gupta, p.147/150	"The above non-mathematical (literary and speculative) use of Śūnya (or zero) may be said to form the first or the earliest phase of the technology of using zero in India. Subsequently there were semi-mathematical and mathematical uses of the concept of zero." "The Buddhist philosophy of Śūnyavada ('Zeroism') springs from the doctrine of non-existence of any Spirit either Supreme or human. The saying sarvam Śūnyam (all is void) is attributed to the Buddhist Nāgārjuna (about A.D. 150). One of the names of Brahma is Śūnya…Simple arithmetical operations involving zero were known and carried out in India earlier than the time of Brahmagupta (A.D. 7th century)…But Brahmaguota seems to be the first in the world to give a formal exposition of the Śūnya-ganita (mathematics of zero)…The fourth of these seven rules may be taken to be Brahmagupta's definition of zero as given by

	P + (-P) = 0
Vedic Numerical System including Śūnya, by S.A.S Sarma, p.33	"The concept of maya and void in the Vedic philosophy, the Śūnyavada theory of Buddhists propounded by Nagasena and Nāgārjuna and the abhava of Nyaya system are all quoted for the genesis of zero."
Brahmi Numerals and Decimal Notation: Nature and Evolution, by Ajay Mitra Shastri, p.74/79	"The naivety of this system [i.e. Brahmi numerical system, PG] were soon realized, and some unknown genius invented the zero or Śūnya which made the arithmetical calculations extremely simple."
Reflections on Pāṇinian Zero, by M.D. Pandit, p. 116/118	"If we look at possible parallels of the technique in other sciences, we find that the sciences which represent or employ a technique parallel to the Pāṇinian zero are three. They are: the Buddhist philosophy of Śūnyavada; the Nyaya-Vaisesika concept of abhava and the technique of zero in mathematics…the zero as a number, preceding the positive integer 1, can roughly be compared with the Śūnya of the Buddhistic philosopher Nagasena. Just as Nagasena arrives at zero by negating or subtracting all he parts of a thing, the mathematical zero can also be arrived at by subtracting a quantity from itself…The same procedure is visible in getting the number zero in mathematics. To illustrate, the number 0 = 1-1, or generally x-x. The number zero in mathematics can thus easily be compared with Śūnya of the Buddhistic philosophy of Śūnyavada, since the procedures to arrive at Śūnya/zero are the same in both sciences. The mathematical zero as a marker for a vacant place in numbers like 10, 20, 30 etc. however cannot be compared with the Buddhistic Śūnya, since the Buddhistic Śūnyavada has no procedure similar to the one in the mathematical case."
The Importance of the Concept of Zero in Modern Mathematics and Science, by J.N. Kapur, p184/185	"He knew that it was not a counting number and he called it Śūnya, but as soon as he got Śūnya, he wanted to have a number preceding Śūnya and then in the same way invented the integers -1, -2, -3…and now he felt a degree of satisfaction with this new system of integers…He realized for the first

	time his freedom to create objects without worrying about these objects having a correspondence with objects in his physical experience. It was this freedom to create which was even more responsible for the development of mathematics and civilization that the creation of the number 0 itself. "Another way in which he could have arrived at zero and negative integers was through the operation of subtraction. In the system of natural numbers, he could subtract a smaller number from a larger number, but there was some feeling of incompleteness since he could not subtract a larger number from a smaller number or a number from itself. He invented integers in order to achieve a sense of completeness. He had no other alternative, but to invent negative integers and 0. Again he realized his freedom to create new mathematical objects in this manner."
Impact of Indian Philosophy on Bhaskariya Concept of Zero, by A. Mukhopadhyaya, 197/202	"According to Nāgārjuna's philosophy, the primary meaning of Śūnyatā is the sense of devoidness, which is a direct reference to the truth of things mundane (worldly) and ultimate, but refers also to the method by which Śūnyatā as truth is brought to light by rejecting the imagination of the ultimacy and absoluteness of particular entities and of specific concepts and conceptual systems…The ultimate nature is devoid of origin and is impermanent...The mathematical definition of Śūnya (zero) (dhanarnayor ntarama samaikyakham, i.e. the sum of two equal quantities differing in signs, is zero), as was introduced by Brahmagupta, follows the idea of abhava in the sense of abandoning a number which was existent before (prag, utpatteh paratsa catmaprananna bhavati). The concept of Śūnya was clarified as sunamĀkāśam where the words Śūnyam (zero) and Ākāśam (the sky) were synonymous…According to Nāgārjuna philosophy, Ākāśa is a name, a permanent principle of accommodation without resistance, it is formless and without

	any specific character. It is the same nature as nirvana, the ultimate goal to which all beings tend for fulfilment..It is not clear whether Brahmagupta's concept of Ākāśa had any bearing on the above two philosophical doctrines…That the mathematical concept of Śūnya in India is an outcome of Indian philosophical thought, will be clear if we think how the philosophy of abava had been almost a traditional guiding philosophy behind the pre-Bhaskariya concept of Śūnya. So, it is quite probable that Bhāskara II's concept of Śūnya was built up against the background of Nāgārjuna's philosophy."
Philosophical and Mathematical Implications of Zero in Indian culture, by S.A. Paramahans, p.209	"To conclude, we see that the development of the concept of zero and its form in India were related to the form of the Brahmi sign for 10. We have identified a late form of Brahmi 10 as the immediate parent of the zero sign."
Concept of Śūnya and Sakta Tantras, by M.C. Joshi, p.218	"However, more significant is the employment of Śūnya in Indian philosophical systems particularly by Mahāyānic Buddhists and Sakta teachers. A significant point which deserves to be noted in regard to the transformation of the property of Śūnya in arithmetic and the Indian though is the conceptual change from quantitative to qualitative proportions.
Number Mysticism in Other Regions and the Impact of Śūnya, by P. Manansala, p.239/45	"The ideas of non-duality, monism, Śūnyatā and similar concepts were well-suited, in regard to number mysticism. To the numeral Śūnya, or zero. Whether such philosophies, which predated the first recorded instances of the use of Śūnya, had any influence on the development of the latter is beyond the scope of this work. Certainly it is possible, but whether the case, the creation of a symbol for Śūnya could not have found a better suited environment in which to arise. Śūnya was the missing piece in Eastern number mysticism…The numeral zero also represents that from which all numbers arise, and by negation, that to which all numbers return…The positive and negative forces , represented by the sets of positive

	and negative numbers both have their starting point at Śūnya. When united, these sets negate each other and return to Śūnya/zero…In Buddhism, negation of the self leads to nirvana, which itself means to extinguish a flame'. The self minus the self comes to equal the Ultimate Reality (1-1=0), known as Śūnyatā."
Indian Palaeography, by A.H. Dani, Munshiram Manoharlal Publishers, New Delhi (1986), p.xiii/xvi	"The decimalising process was a deliberate invention of a mathematical genius that could not have taken place before the value of zero (Bindu or Śūnya) was understood. I believe that this was a logical development of the Buddhist philosophy of Śūnyavada, when a new meaning was given to it about fifth century A.D. Here the new meaning transcends the dead zero of the earlier period to a living potential cypher.
The Concept of Emptiness (Śūnyatā) in Modern Mathematics, by Ankur Barua, N. Testerman, M. Basilio, Buddhist Door, Tung Lin Kok Yuen Hong Kong, 2009	Since, Nāgārjuna's Doctrine of Emptiness or Śūnyatā was quite popular in Indian society during the time of Brahmagupta, there is a high probability that Brahmagupta was inspired by this Doctrine of Emptiness. Thus, the philosophical concept of 'emptiness' or 'śūnyatā' or 'void' gave rise to the concept of 'zero' in Indian mathematics. Subsequently, this became the foundation for modern mathematics.

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